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Arithmetic Geometry

from Keio's Faculty of Science and Technology

Approaching problems in number theory using geometric intuition

Kenichi Bannai

Associate Professor
Department of Mathematics

Featured in this issue is Associate Professor Kenichi Bannai, a researcher in arithmetic geometry – a field of mathematics – tackling unsolved problems in number theory via an approach based on geometry.

Bringing new insight to mathematical conjectures

Balance between logic and intuition

The combination of logic and intuition is a powerful tool in solving many problems. Arithmetic geometry is a field which uses geometric intuition to solve problems in number theory – intuition which may come to you, for example, when you look at geometric figures comprising of points, lines, etc. We paid a visit to Associate Professor Bannai, who is researching problems in number theory – related to properties of integers – using methods of arithmetic geometric.

A delicate balance between logic and intuition

"When trying to solve a problem in geometry, one may have had the experience of being able to solve it easily by drawing an additional line in the original figure – an approach which stimulates intuition. Arithmetic geometry approaches problems in number theory using concepts of geometry as represented by such figures," Dr. Bannai outlined his field of study.

What do we mean by geometry? Let's take for the example the problem of finding all the rational number solutions of the equation: $x^2 + y^2 = 1$. You may of course solve this problem by manipulating the equation. On the other hand, the solution becomes much easier to understand if you think of the equation as describing a unit circle with a radius 1 on a plane (see Fig. 1). (x, y) = (-1, 0) is a rational number solution of this equation. Consider a straight line passing through the point (-1, 0); this line meets the unit circle only at one more point other than (-1, 0). If the slope of this line is a rational number, then the point of intersection provides a rational number solution of the equation. Conversely, if a rational number solution of the equation is given, then if we connect this point to (-1,0), then we obtain a line through (-1, 0) with rational slope. This implies that the rational number solution to the original equation corresponds one to one to the lines through (-1,0) whose slope is a rational number.

Arithmetic geometry, being useful in solving number theoretical problems, has developed alongside number theory. Number theory has a very long history. Even a simple problem concerning integers in many cases has behind it highly sophisticated mathematical theory. Number theory also has many real-word applications; it has been applied to cryptography and authentication technology. As an approach to solving problems in number theory, arithmetic geometry is expected to contribute to solving many problems. In fact, the methods were used to prove Fermat's last theorem and Mordell's conjecture(*1).

"What I've pursued so far in my research are arithmetic geometric functions known as the polylogarithm. The polylogarithm is related to many important invariants in number theory. There are certain conjectures in number theory that predicts the relationship between two number theoretical invariants which a priori do not appear to be related to each other. If we can present a higher mathematical concept that can explain both invariants in a unified manner, it will help us get one step closer to shedding light on their relationship. The polylogarithm can serve as a useful bridge between them," remarks Dr. Bannai.

Finding relationships between important invariants by capturing the essence of the problem

The polylogarithm, on which Dr. Bannai focuses, is a kind of which we call a "geometric object." To better explain its concept, we interpret a "geometric object" as some sort of "figure." If one has a figure, it is possible to derive various invariants, such as the "area" and "the number of vertex." Therefore, suppose there are several invariants

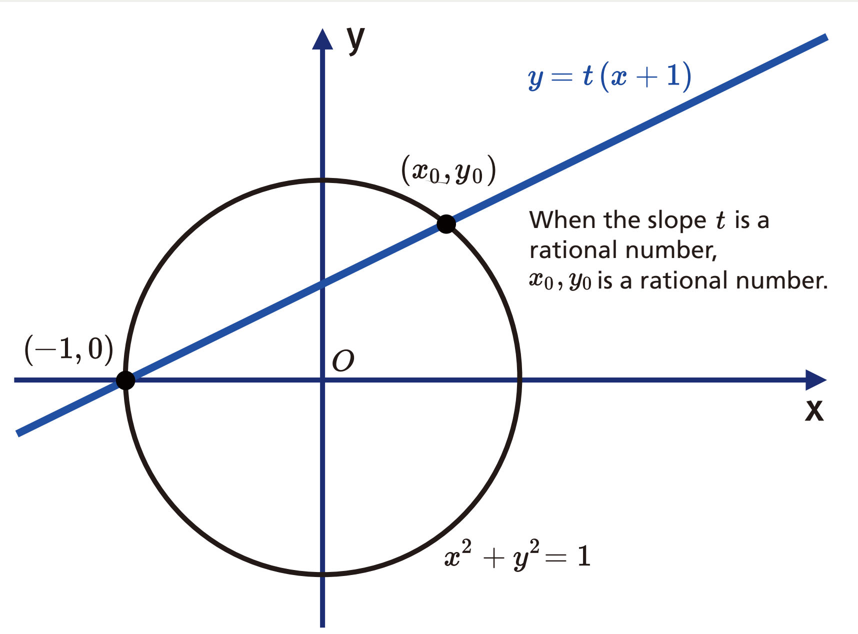

Fig.1 Solving problems in number theory using geometry
A rational number solution for the equation $x^2 + y^2 = 1$ corresponds one to one to a line through the point (-1, 0) with rational slope. The solution is easily visualized if one uses geometry and views the equation as defining a unit circle.

which you wish to compare. If one could find a figure that intrinsically contains these invariants, then this figure would be the starting point for finding some relationship between the various invariants. In a sense, the polylogarithm is a figure that intrinsically contains important invariants in number theory.

"Suppose you can represent an invariant 'A' as a simple figure by abstracting its properties. Using the same technique, one next studies another invariant 'B' which looks completely different from 'A.' If one can express the invariant 'B' using the same figure as that of 'A,' then we can hope to find a formula or rule that governs both A and B." Alexander Grothendieck(*2), who contributed greatly to algebraic geometry, referred to such geometric objects obtained through abstraction as "motives." The polylogarithm is an important example of a "motive," which plays a vital role in arithmetic geometry (Fig. 2).

Dr. Bannai adds, "The term 'motive' used by Grothendieck comes from a commentary of the work of Paul Cézanne(*3). The post-impressionist painter Cézanne distanced himself from impressionists who depicted objects using the ever changing quality of light and shadow. Cézanne instead establish his own style emphasizing the "vital intensity that the motif possessed in its actual existence." The 'motive' in arithmetic geometry also sees through the surface and captures the true essence."

Solving mathematical problems through cooperation

In stark contrast to the depth of his research, the reason Dr. Bannai chose the polylogarithm as his research topic seems very simple.

"I chose this theme because I was attracted by its simplicity," says Dr. Bannai smilingly. One may imagine mathematicians as working alone when doing research. In reality, this is not the case.

"Remembering struggling through mathematics in high school, one may have the impression that mathematical research is a solitary endeavor. However, in reality, mathematical research typically advances through discussions among mathematicians around the world. Mathematics is a highly internationalized field of study. For example, my recent series of work is a joint project with Prof. G. Kings of Regensburg University in Germany. The situation is the same in my lab. While everyone has their own theme, members advance their studies through active discussions. They are taking advantage of being in a group and are enjoying its benefits."

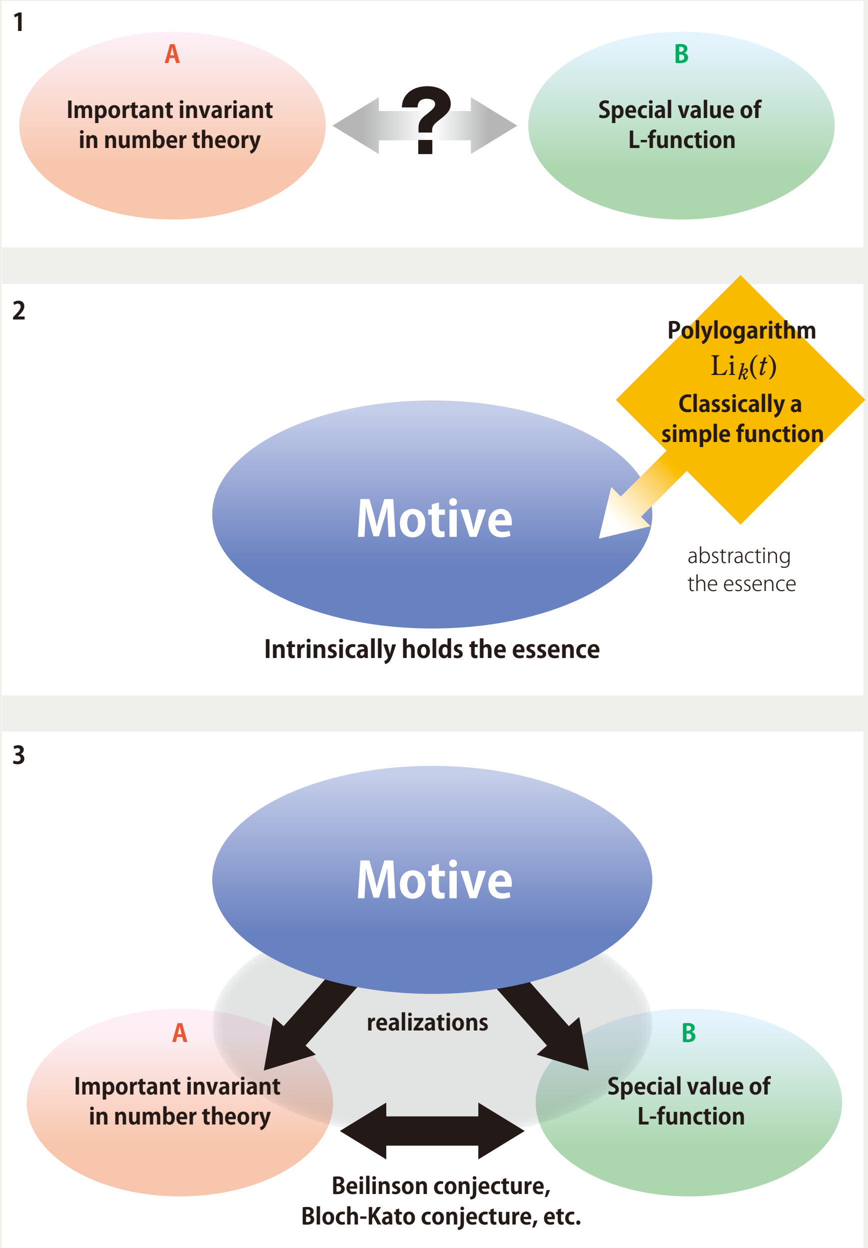

Fig. 2 Polylogarithm at work

(1) We consider "An important invariant in number theory" and "The special value of an L-function" – two invariants which appears unrelated to each other. (2)By starting from and abstracting the polylogarithm function, we construct a "motive" which gives a higher conceptual interpretation of the two invariants. (3) By going through the "motive," we are able to find the relationship between two invariants which at first seemed unrelated. Such relationships may derive solutions to problems, such as the Beilinson conjecture and the Bloch-Kato conjecture.

Dr. Bannai believes that polylogarithm will play a key role in understanding important problems in number theory. Above all, he is fascinated by the idea of finding unified rules through abstraction of various properties and quantities. He dreams that such methods would give clues to solving important conjectures.

(Reporter & text writer: Kaoru Watanabe)

＊1 **Fermat's last theorem and Mordell's conjecture:** Both had long been unsolved mathematical problems, but Mordell's conjecture was solved in 1983 by Gerd Faltings, and Fermat's last theorem was solved in 1995 by Andrew Wiles.

＊2 **Grothendieck:** Alexander Grothendieck is a founding member of the Institute of Advanced Scientific Studies (IHÉS) of France. Working on the foundation of algebraic geometry, he expanded and enriched its horizon by bringing in commutative ring theory and various other mathematical methods –revolutionizing the world of mathematics.

＊3 **Cézanne:** Paul Cézanne is a post-impressionist painter. Originally an impressionist, Cézanne gradually came to realize the importance of capturing the "motif." He had great impact on 20th-century art.

Expanding the world of mathematics

Connecting different fields through abstraction

Kenichi Bannai in interested in solving problems in number theory using a geometric approach. He spent most of his childhood in the United States before permanently returning to Japan in high school. It was during this period that he became increasingly fascinated by mathematics. He loves mathematical speculation so much that it has now become his major "hobby." Bannai has found in mathematics the capacity that could guide our relationships with society or with other members of community. Based on this experience, he aims to promote mathematics in a more open way.

You spent your childhood in the United States. Since when?

Since when I was 2-years old. Both of my parents are mathematicians, and our family moved to the States in the mid 70's when my father was offered a position at the Ohio State University. Except for two years of kindergarten, I lived in the States until I returned permanently to Japan near the end of high school.

Growing up in the States, did you consider going to an American university?

At the time when my family was thinking about returning to Japan, I had graduated early from high school and received offers from several American universities to attend. It was in the late 80's, when the world was feeling threatened by the rapid rise of the Japanese economy. Japan was portrayed as a country of people who were forced to work like machines all year around. Viewing repeated TV footage showing Japanese employees exercising in unison, I received the depressing impression that Japan was a country without freedom or individuality.

Then one day, I encountered Hayao Miyazaki's film "Nausicaä of the Valley of the Wind." It was a fantastic visual depiction of an ecosystem totally different from our world, and I was totally overwhelmed by the force of imagination that could create such a world. This experience deepened my interest in Japan, and influenced my decision to return to Japan with my family.

Why did you become interested in mathematics?

I started seriously thinking about mathematics in high school. Then, something happened which made me think that mathematics may be good.

It happened during a class in chemistry; I came across a differential equation describing the change in concentration of a substance during a chemical reaction. It was an equation I had just learned during a class in mathematics, but at first, I did not think much of it. Then immediately after, during a university biology class that I attended, when calculating the growth of a population of an animal in a certain ecosystem, I again encountered the exact same differential equation. "Wow!" I thought. Furthermore, I saw the same equation being used in an economics class. In other words, I found the exact same abstract

Mathematics has the capacity to abstract things and draw out their essences. Abstraction and other mathematical approaches to problem solving will become an important tool in solving real world problems.

Kenichi Bannai

Kenichi Bannai is a researcher in arithmetic geometry, a field of mathematics which uses geometric intuition to solve problems in number theory. Using arithmetic geometric objects such as the polylogarithm, he has been studying the relation between abstract arithmetic geometric objects and special functions, with an eye towards solving conjectures concerning special values of L-functions. He received his doctoral degree in 2000 at the Graduate School of Mathematical Sciences, the University of Tokyo. The following year, he became an Assistant Professor at the Graduate School of Mathematics, Nagoya University. He visited the École Normale Supérieure in Paris from 2005 to 2007 as a JSPS Postdoctoral Fellow. In 2008, he joined the Faculty of Science and Technology at Keio University. He was promoted to associate professor in 2012.

mathematical equation used with a specific meaning in a wide range of fields. I was convinced that mathematics would be useful in doing anything.

You said you noticed that mathematics was being used in many fields. What do you think now that you are an active mathematician?

Mathematics has the capacity to abstract and draw out the important essence of things and events. I believe that in this increasingly complex modern society, mathematics can play a major role by using its power of abstraction. For example, there are situations where people have so many different opinions that it may seem impossible to come to an agreement. However, if we think deeply about what each person wants to achieve, we may realize that in fact, the goals are actually very similar. Even in cases where specific methods or situations are so different that negotiation seems impossible, if we capture the problem in a sufficiently abstract way, we could achieve a common ground and begin to discuss how to solve the problem in a calm and rational manner.

Although mathematics itself mainly deals with such objects as equations and geometric figures, I am beginning to realize that the mathematical approach to problem solving, including the method of abstraction, is potentially a powerful instrument in solving real world problems.

What do you do when you are not doing research?

I've always enjoyed spending time with my wife. After the birth of my daughter two years ago, my family life is even more fulfilling. However, I don't feel that time with my family is separate from the time I spend doing mathematics. I feel I'm using my brain in the same way – when engaging in research, dealing with my students, or enjoying playing with my daughter. Dealing equally with mathematics and my dearest daughter may sound as though I am a bit too cold as a father, but I feel that I have the same great affection for mathematical issues as I do for my daughter.

Thinking about the essentials of how to associate with or how to establish proper relationships with people has become so much fun that having such an abstract perspective or a mindset based on mathematical speculation is now almost a hobby for me.

What do you think are good points of Keio University?

First of all, the students are very highly motivated. While I was staying in France in 2007, I had the opportunity to participate in a number theory workshop known as the UK-Japan Winter School jointly sponsored by Cambridge and Keio universities. I was very impressed with the enthusiasm of Keio students.

At many Japanese universities, the department of mathematics belongs to the faculty of science. Furthermore, at the University of Tokyo where I studied and Nagoya University where I previously worked, Mathematics belonged to an independent graduate school, which made me feel somewhat isolated from other departments. At Keio University, the Department of Mathematics belongs to the Faculty of Science and Technology, allowing us to be in close contact with the technological departments. I have many opportunities to discuss with faculty from other departments – an advantage creating a health synergy between science and technology. Such a campus environment is probably one reason we have many creative and lively students.

◎ Some words from students . . . ◎

● Prof. Bannai is free of the stereotypical image of a mathematician. He is always willing to advise us even on simple questions and is open-minded enough to take interest in what we say or ask. This broad-mindedness makes his character very attractive. During lectures, he is very enthusiastic and inspires us to study mathematics. When we become stuck with a problem, he kindly gives us hints, often in a subtle manner, to help us along.

(Reporter & text writer: Kaoru Watanabe)

For the full text of this interview
http://www.st.keio.ac.jp/kyurizukai

Lab's workshop

In our lab, we hold a workshop every year. Members of our lab including new students and postdocs participate and lecture about the latest results of their studies.

Hamster – our lab's mascot

The mascot of our lab is the "ordinary hamster." It was originally an illustration by my wife, but we eventually made a stuffed doll.

Seminar scene

This is a scene from our lab's undergraduate seminar. Each student is assigned to read a particular section of a text book on mathematics and then explains the content during the seminar. At our lab, we capture the seminar on video so that students can review it afterwards.

The Bannai Family and Bannai Lab

When spending time with my lab's postdocs and students, as well as my own family, I always try to create an environment so that everyone, including myself, can continue to enrich ourselves.

Side by side in a career in research

My wife is a researcher specializing in neuroscience. We first met when we were freshmen in undergraduate, and I fell in love at first sight. We got married immediately before entering graduate school. Since then, we've advanced our research career side by side.

Weekends with my daughter

I enjoy my weekends spending time with my 2-year-old daughter. Seeing her growing rapidly day by day gives me thoughts about educating students as well as developing myself.

My family, my advisers

Although we study in different fields, my wife and I have common problems which we face as researchers. In this respect, my wife is my best adviser. I'm hoping eventually my daughter could also join our conversation.

私の本棚

My favorite books

Nausicaä of the Valley of the Wind

Feeling threatened by Japan's fast growing economy, the United States during the 80's typically depicted Japan as "a country of imitators without creativity," "a country driving its people into underpaid hard work" and "a totalitarian country where people are forced to work like robots." Under such influence, I myself did not have a positive view of Japan, the country of my parents. When I had to decide between returning to Japan or entering an American university, I encountered the film version of Nausicaä. I was overwhelmed and felt a strong desire to experience living in the same country as the creator of such a film.

Essence of Failure

I was told of the experience of my grandfather who was drafted by the Japanese Imperial Army and experienced many terrible things in China and Burma. After attending elementary school in the States, my classmates repeatedly made a point that Japan was defeated by the United States during the war. Such experiences led me to think deeply about the reasons of what had happened. I think it is very important to learn from past mistakes in doing anything.

The Glass Bead Game

This novel is a biography of Josef Knecht, a fictitious character living in a future world; a master of the "Glass Bead Game." The description of the central device of this book, the "Glass Bead Game" – an artistic game integrating music, the arts and mathematics – awoke my interest in seriously studying mathematic. Although at first I did not take much notice, I recently began to contemplate on the meaning of the story's shocking finale.

Maintaining a Flexible Mind

This book I received as a memento from Shuntaro Sato, a professional conductor of an orchestra with whom we became friends while staying in France from 2005 to 2007. It contains dialogues between Seiji Ozawa (a conductor) and Heisuke Hironaka (a mathematician). Through my relation with Mr. Sato, I was able to confirm that music and mathematics, though seemingly different, have much in common.

Mindset

It has long been my belief that our "talents" and "abilities" can be developed with effort. This famous book, based on rigorous psychological experiments, explain that one can obtain positive results by maintaining such a mindset. I hope to be able to convey to my students that such a positive mindset is very helpful in future success.

Realization of Polylogarithms

In this paper, the author Jörg Wildeshaus constructs the realization of the polylogarithms in a very general setting. After attending an intensive lecture by Prof. Kazuya Kato (now a professor at University of Chicago) in graduate school, I learned the difficulty of constructing motivic objects. Immediately after, I encountered this book which suggested that the construction of the polylogarithm was a promising method to construct important motivic objects in a very general setting. This experience convinced me to make polylogarithm as my central topic of research.

Stories of Mathematics

This series is an introductory account of mathematical concepts that high school and college students will encounter. The books provide students with a good overview of mathematical concepts before they jump into more technical books. The book is recommended to high school students curious about what lies ahead.

Mathematics and Individual Talents

Kenichi Bannai

I often hear people say that mathematics is difficult or that they cannot figure out how to solve mathematical problems. I understand completely. As a mathematician, I also daily face mathematical phenomena and problems that I'm not sure I would be able to understand or solve. Nevertheless, if I keep trying to understand or struggle for a solution, the truth begins to reveal itself a little by little. As in the famous quote "There is no royal road to mathematics" by Euclid, it is not possible to acquire mathematical ability overnight.

When thinking about mathematical geniuses, one may imagine someone with inborn talents who can immediately understand anything mathematical without any effort. However, as Terence Tao, who was awarded the Fields Medal for his overwhelming achievements, wrote in his blog, mathematical ability may be greatly enhanced through effort. Just as the skill of world-class athletes and musicians are made possible by their tireless daily effort, the ingenious ideas by mathematician to find solutions to problems also result from daily effort.

When encountering a totally new concept or an extremely difficult problem, any mathematicians, no matter how talented, need to take time and think deeply. Just like a muscle, the power to think can greatly improve by applying a heavy load. If you carelessly think that you may not have talent, you may prematurely give up in a stage where everyone finds things difficult.

For example in the past, many people mistakenly believed that women had lower mathematical ability. I would imagine there were women who were discouraged and gave up learning mathematics. However, due to advancement in education, the difference in the US of distributions of scores in a standard mathematics test for male and female high school students. This indicates many women were able to free themselves from bias concerning mathematical ability.

One feels better when after physical training and your body is more fit. Likewise, it is very pleasant to find yourself with improved ability to think after persistent effort. Mathematics requires effort to understand, but this is what makes the world which opens up the more exciting!

Science and Technology **Information**

The 16th KLL Industry-Academia Collaboration Seminar

http://www.kll.keio.ac.jp/
Date: February 22 (Fri.), 2013

This seminar will introduce our research activities related to "precision machining", "actuation" and "simulation" which are the core themes of mechanical engineering. Details of this seminar will be published on the above website.

Construction work starts on the New #34 Building (tentative name) of the Faculty of Science and Technology (on Yagami Campus)

On December 12, 2012, the groundbreaking ceremony was held for the construction of the New #34 Building (tentative name) of the Faculty of Science and Technology (on the Yagami Campus) as one of Keio's 150th anniversary projects. The new building is expected to serve as an educational venue of practical experiments for students in the mechanical, chemical and administrative engineering departments as well as an ideal environment to nurture top-notch talents in our science and technology fields. The building is scheduled for completion in January 2014 – the year our Faculty of Science and Technology will celebrate its 75th anniversary.

Artist's concept of the building upon completion.
(The white building in the upper part of the visual on the left)

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Editor's postscript

The composition of Mr. Bannai's portrait on the front cover was decided by himself wishing to create an image of Mr. Bannai ready to talk with individual readers of this issue of "New Kyurizukai." During the actual photographing, we had one of his lab's students stand behind the camera and keep talking with Mr. Bannai while shooting. This indicates just how Mr. Bannai values dialogues with others and wishes to visualize his intention in the photo.

For the interview and photographing, he appeared wearing a light blue shirt and began talking in a mild tone and with gentle smiles. While it is customary to wear suits at academic meetings for engineering, he told us that participants at scientific society meetings would regard such clothes as out of place – a difference between the two academic societies.

Beginning with this issue, I (Ms. Yuko Nakano) have become in charge of this bulletin and look forward to meeting a number of up-and-coming research scientists. I am determined to do my best so that we can deliver information about the attractiveness of these researchers' themes as well as their individual characters.

(Yuko Nakano)

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