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4-Dimensional Topology

from Keio's Faculty of Science and Technology

Understanding 4-dimensional space
by focusing on singularities

Kenta Hayano

Associate Professor
Department of Mathematics

This feature covers Associate Professor Kenta Hayano, who studies figures called "4-manifolds" in the field of topology, a modern branch of mathematics.

"Topology's front lines," a highlight of modern geometry

Exploring invisible "4-dimensional manifolds"

The Earth is round. This is now a well-known fact. However, it was not until the 20th century, when rockets were invented, that it became possible to see the entire globe. Currently, we do not have a complete picture of the universe. This is because we cannot observe it from the outside. We may, however, be able to solve this problem through mathematical theory. This is where the field of "topology" comes into play.

What is a "manifold" in topology?

Topology is a branch of geometry. The geometry we learn through high school is called "Euclidean geometry," but topology, a field broached in college-level courses, classifies figures and examines their features in a different way.

Topology deals with figures called "manifolds." A manifold is defined as "a figure or space in which a part, but not the whole, can be perceived." The surface of the earth is a "2-dimensional manifold" or "2-manifold" in the topological world. This is because when viewing very small portions of the Earth's surface, it is possible to perceive it as a "flat," two-dimensional space. It's easy to think that the surface of the Earth is 3-dimensional since we often depict it using 3D space. However, a better way of phrasing this concept is that the Earth's surface exists within 3-dimensions. Because we can map the points comprising the surface of a sphere using two independent parameters (x, y), the surface is actually 2-dimensional.

Visualizing unimaginable "4-manifolds"

Topology deals with even higher "n-dimensional manifolds" such as "3-dimensional," "4-dimensional," and "5-dimensional" manifolds. Experts call manifolds with 4 dimensions or less, "low-dimensional manifolds," and those with 5 dimensions or more, "high-dimensional manifolds."

Among them, Hayano studies 4-dimensional manifolds, more commonly called "4-manifolds." A 4-dimensional manifold is a manifold in which each of the points that comprise the figure are represented by four independent parameters. Just as there are an infinite number of 2-manifolds, there are an infinite number of 4-manifolds. However, since we can only perceive up to 3-dimensional space, we cannot imagine or physically see what a 4-manifold is.

"Previous studies have developed methods of visualizing the characteristics of 4-manifolds using diagrams. One of them is the 'Kirby diagram,'" says Hayano.

A Kirby diagram for a 4-manifold looks like what is shown in Figure 1. When you have a specific 4-manifold you're working with, an important factor in understanding its features is studying singularities of functions defined on the

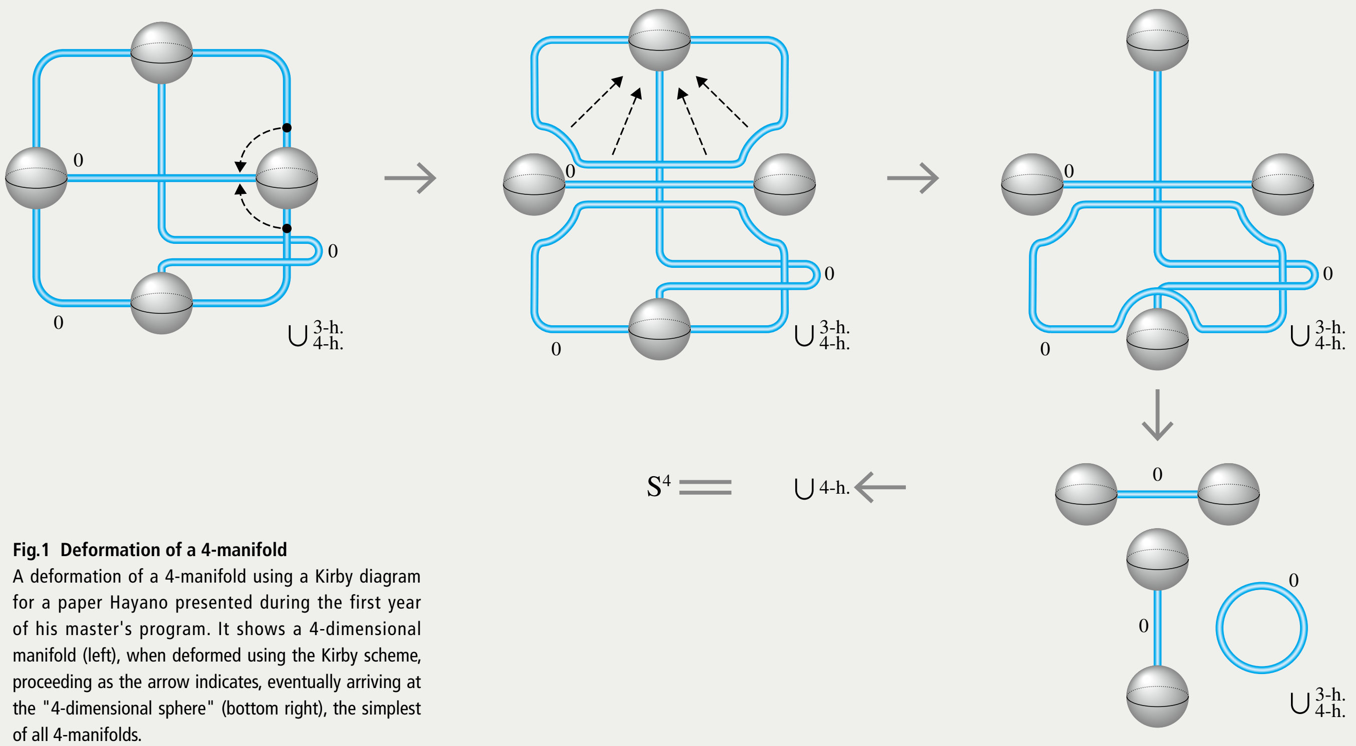


Fig.1 Deformation of a 4-manifold
A deformation of a 4-manifold using a Kirby diagram for a paper Hayano presented during the first year of his master's program. It shows a 4-dimensional manifold (left), when deformed using the Kirby scheme, proceeding as the arrow indicates, eventually arriving at the "4-dimensional sphere" (bottom right), the simplest of all 4-manifolds.

Fig.2 Illustrating the figure of a 4-dimensional manifold
In a technique known as "trisection," the position of singularities of a function on a 4-dimensional manifold is shown using three figures.

4-manifold.

"You can work out the shape of a manifold by examining the singularities of a function and other methods. A singularity is the point at which the derivative of the function is zero. A Kirby diagram follows certain rules to capture 4-manifolds by illustrating the positional relationships between the singularities of their functions," Hayano explains. He posits that while the average person is incapable of picturing the shape of a 4-dimensional manifold, topologists can by simply looking at a Kirby diagram. It's worth noting that there are other methods to use diagrams to visualize the features of 4-manifolds. For example, Figure 2 illustrates the "trisection" technique, in which the location of singularities of a function on a single 4-manifold is represented by three schematic diagrams.

Hayano says that he studies various 4-manifolds and discusses their characteristics with his students on a daily basis.

4-manifolds; a major development in the "gauge theory" of quantum mechanics

"I was very interested in the fact that with 4-manifolds, there is a big difference between 'topological structure' and 'differential structure,'" says Hayano.

"A topological structure is a structure defined solely by the way each point in the manifold is connected. On the other hand, differential structure, which looks at the form's 'smoothness' to classify manifolds in further detail. Differential structure is a type of topological structure that contains more information about the shape of the manifold," he explains.

In Euclidean geometry, which students learn up through high school, figures are classified according to the lengths and angles of their sides. In the field of topology, Hayano describes how he is "motivated to come up with structures for manifolds that posit additional information, much like differential structure, and further classify them accordingly."

"In the world of topology, people say that with 2-manifolds 'A cup of coffee and a doughnut are the same thing.' Both have a single hole, and when deformed (flattened) they become the same shape. This classification is based on the topological structure of the manifold that focuses on the how the points that make up the figure are connected. Unfortunately, the difference between topological structure and differential structure does not appear in manifolds with 3 or fewer dimensions, so humans cannot really experience it, but in 4-manifolds, the difference between topological structure and differential structure is remarkable," Hayano explains.

He is also interested in the "complex structure" of 4-manifolds. "If a manifold has a complex structure, it will have both topological and differential structure. The opposite is not always true. Just because a manifold has topological structure does not mean it has differential structure, and just because it has differential structure does not mean it has complex structure. Among 4-dimensional manifolds, two can share the same topological structure but have different differential structures. You can also have two differential structures for the same manifold, with one obtained from complex structure but not the other. It's these types of details that really attract me to researching 4-manifolds," Hayano explains.

The existence of 4-dimensional manifolds that have topological structure but lack differential structure comes from the Rokhlin's theorem, which was proved in 1952. It was again proven in 1982 by British mathematician Simon Donaldson (1957-present), who used the gauge theory of quantum mechanics to demonstrate that 4-manifolds with multiple different differential structures exist. This led to major developments in the study of 4-dimensional manifolds.

Will we ever know the "shape" of the universe?

You may be wondering what the point of studying n-dimensional manifolds is.

According to Hayano, "You can't untangle two intersecting lines on a flat, 2-dimensional plane. However, when expanded to 3-dimensional space, the two lines can be distinguished when crossing below or above the other, making it easy to understand the intersection. As you increase in dimension, you have more freedom to move the manifold, which makes it easier to perform operations such as untangling intersections, therefore making the manifold easier to work with."

In fact, cutting-edge physics research hypothesizes that "higher-dimensional spaces" exist beyond the 4th dimension while string theory suggests that the universe may be 10-dimensional if the dimension of time is included. Therefore, topology, which deals with higher dimensions, has become an important mode of research.

It may not be long before Hayano and other topologists are able to solve the mystery of the shape of the universe.

(Interview and text writer: Kumi Yamada)

The key to enjoying mathematics is to let your interests lead you when an intriguing problem presents itself.

According to Hayano, he has had a natural talent for arithmetic and mathematics since childhood. He first learned that math could be a career in his first year of junior high school and set his sights on becoming a mathematician. Wary about whether he would have what it takes to become a full-fledged mathematician, he started full-scale research early on. This led to him earning a Ph.D. in the first year of his doctoral program. We asked him to tell us his secret to sticking with and enjoying mathematics for so long.

What made you decide to become a mathematician?

When I was in the first year of junior high school, I watched the TV drama "Yamato Nadeshiko" which is how I learned that you could do math as a profession. I started to think seriously about becoming a mathematician when I entered high school, and decided I would major in mathematics when I got to university. That being said, I did more than just study math in high school. I was also crazy about swimming and spent my summers swimming more than 10 kilometers a day at camp.

Among the areas of mathematics, what made you choose topology?

Actually, I didn't study much during the first two years of my undergraduate program. Instead, I joined a competitive student dance club and spent my all my time practicing for that. But that feeling gave way to anxiety in my junior year when I realized I needed to get serious about math. I quit my extracurriculars and started focusing on my studies. Just around that time, I decided to attend a small mathematics seminar that was starting on campus. The seminar teacher recommended I read a book on topology. The more I read, the more intrigued I became.

I heard that you went on to attend graduate school at Osaka University and even studied in Germany.

I started writing my paper in the first year of my master's program. Right around that time, I learned of a professor who had published a paper dealing with the same topic as mine. Despite researching the same subject, some of our results were different, so I asked my advisor to contact them so we could meet. After our meeting, the co-author of that person's paper invited me to Germany to study for Mathematics for six months while I was a doctoral student.

In your first year as a doctoral student, you were both selected as a Research Fellow of the Japan Society for the Promotion of Science (JSPS) and received your doctoral degree in one year. Does that mean you started working on your dissertation before your program began?

Trying to become a mathematician is a risky move. I was quite anxious as a junior and senior in university when contemplating whether I could support myself in the future if I pursued math as my career. My professors often stressed that study skills and research skills are two different things.

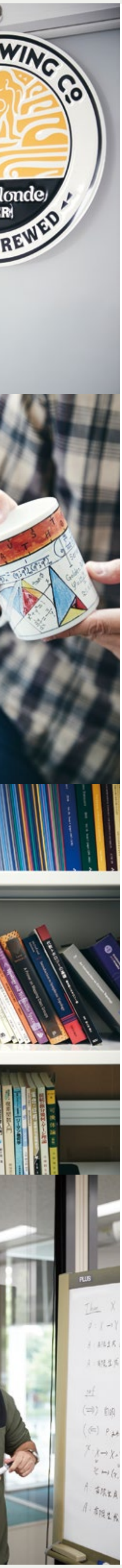
I was confident in my ability to study and learn material as long as I was given time, but I couldn't gauge my aptitude for research. So, I decided to begin research early on to see if I had what it takes to make it as mathematician. From the second half of my senior year, I began to approach my studies with research in mind. By the first year of my master's program, I started a full-fledged research project. There are many different fields within mathematics.

After completing your doctoral program, you taught at Hokkaido University?

I was offered the job after applying for an open position. Hokkaido was great. The food was delicious, and it was perfect for winter sports. The position had a maximum term of 5 years, so rather than waiting for my contract to run out, I took up another open position at Keio University in 2016.

What path do most math majors take after graduation?

Many of them go on to the corporate world or end up becoming math teachers at middle schools and high schools. However, as far as my field is concerned, I'm confident saying that there are almost no companies looking to hire an expert in topology [laughs]. There are some niche opportunities, though. I am currently collaborating with a researcher at a gaming company who is working on automating the development of game software



I dreamed about becoming a mathematician after seeing it on TV when I was a kid. Years later, that dream has become a reality.

Kenta Hayano

Department of Mathematics

Dr. Hayano specializes in low-dimensional topology, mainly studying "4-manifolds" and "singularity theory." He earned a Ph.D. in Science from the Department of Mathematics, Graduate School of Science, Osaka University in 2013. After teaching as an assistant professor at Hokkaido University, he started working as full-time lecturer in April 2016 at the Department of Mathematics, Faculty of Science and Technology, Keio University. In April 2020, he was promoted to Associate Professor (current position).

through artificial intelligence (AI) using "singularity theory," which is also central to the study of 4-manifolds. Machine learning and deep learning with AI are currently being used in many fields, and I feel that there are quite a few areas where singularity theory could prove useful.

Understandably, it may be difficult to picture viable career paths when it comes to mathematics, but students shouldn't have any trouble finding a job. In fact, it seems that more and more companies are saying, "We want to hire people with a Ph.D. in mathematics." Today, there are few companies that have yet to adopt IT. People with a background in mathematics are in great demand across a variety of industries.

What advice would you give to high school students who want to major in mathematics in university?

First of all, anyone who is willing to learn is welcome. Unlike chemistry or biology, mathematics requires no laboratory equipment or chemicals, costs very little, and can be started at any time.

What are your future goals as a mathematician and what are your dreams for the future?

I currently have several unsolved problems that I have been working on for the past 3-4 years. Of course, solving them would be amazing, but I would be perfectly happy if I could simply continue working on them every day. The key to enjoying mathematics is letting your interest guide you to the problems you work on. I don't harbor any secret ambitions or get myself worked up saying things like "I WILL solve X problem in my career if it's the last thing I do!" [laughs]. I would like to keep the focus on having fun while working in mathematics.

◎ **Some words from students . . .** ◎

● In my Junior year, I became fascinated with topology after sitting in on one of Dr. Hayano's lectures. He really had a way of explaining concepts that's easy to understand. I wanted to go more in-depth with topology, so I chose his lab for my research project. His attentiveness in the classroom carries over to his lab. Thanks to his guidance, I can feel myself making steady progress and building up my understanding of the subject matter. (1st-year master's student)

(Interview and text writer: Kumi Yamada)

For the full text of this interview

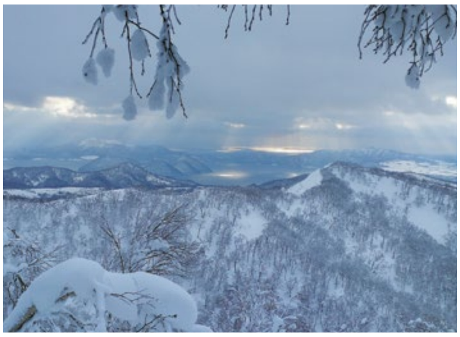
https://www.st.keio.ac.jp/en/kyurizukai/

My farewell party at Hokkaido University

This is a picture taken at my farewell party when I left Hokkaido University. It was held at a bar I used to frequent. The costume and sash were provided by the students who organized the event.

Award Ceremony for the Takebe Katahiro Prize

This is a commemorative photo taken when I received the Takebe Katahiro Prize from the Mathematical Society of Japan in September of 2018. (I'm in the front row, second from the left)



Lake Toya as seen from the Rusutsu ski resort

The slopes of the Rusutsu ski resort face the opposite direction of Lake Toya, so you can't see it from the ski trail unless you sneak a peek from a spot along the ridge just past the summit. I've made the trek to this exact spot many times, but Lake Toya has never looked as beautiful as it did on this occasion. Enjoying views like this is one thing that makes winter sports so fun.

Kenta Hayano's ON and OFF

This part of the feature will focus on Hayano spends his time off rather than what he does on the clock.

At the summit of Chisenupuri

This is a picture taken when I climbed a snow-covered mountain with friends from Sapporo. My face is completely hidden by my helmet, goggles, and balaclava, but I'm the one on the right with a tassel of grass behind my head. After taking this picture I snowboarded from the summit to the bottom of the trail in one go. It felt amazing!

One of my hobbies is trying out lots of different restaurants by barhopping. Here are some of the most memorable dishes that I've encountered.

"My usual" at a craft beer bar I frequent

I like pasta and rice gratin with white sauce and cheese, so its one of my regular orders at the bar. The mac and cheese in the photo is a dish from a bar I frequent. I've ordered it enough that I can simply say "the usual" and they'll serve it to me.

Shishamo (smelt) sushi

I had this when I visited the "Shishamo Festival" held in Mukawa, Hokkaido. What Honshu locals call "shishamo" or smelt is technically capelin, a similar, but different, fish. Real Hokkaido shishamo is quite rare. Even in Hokkaido, shops mostly sell it dried, so this festival presented a rare opportunity to eat it while fresh.

The most delicious potato salad

Potato salad is one of my favorite foods. It's a simple dish, but one that I think says a lot about an establishment, so I always like to try it. This place's potato salad is one of the best I have ever had. It is quite an elaborate dish with smoked potatoes, bacon, poached eggs, nuts, corn, and garnished with cumin and gorgonzola.

Wild boar sashimi

I had sliced raw wild boar at a local Japanese pub while on a business trip to OIST (Okinawa Institute of Science and Technology Graduate University). I heard that it important to eat wild boar sashimi while it's still fresh. This restaurant is the only place on the Okinawa's main island where you can get it.

A huge paella

This enormous paella was served at the lunch on an excursion for a research meeting held in Valencia, Spain. You can tell how large it is just by comparing it to the surrounding chairs and spoons.

私の本棚

My favorite books

Splatoon 2

I've been playing Splatoon since the first game came out on WiiU. Skill-wise I'm S+, and have reached Rank X several times in the Tower Control and Rainmaker battle modes. However, I find it hard to stay ranked X, and quickly get knocked back down to S+ (those who play will understand). I'm not a beginner, but also not that good either. Each match is short, lasting about 5 minutes as most, so I sometimes play a few matches as a break from research at home.

4-Manifolds and Kirby Calculus

This is an exhaustive textbook covering Kirby diagrams. I read some of it in my thesis seminar when I was a master's student. There are 558 pages. I still haven't read all of it, but sometimes I reference it in place of a dictionary in my research.

4-dimensional Topology

This book explains the motivation for focusing on 4-dimensional topology. The book is carefully explained from the basics, stating in the preface that "the reader is not assumed to have any prior knowledge other than a high school level of mathematics." That being said, I think it would be difficult for a high school student to grasp all of the details the book covers, but I think it is the perfect start to get a rough idea of what kind of field 4-dimensional topology is.

Algebraic Geometry I. Complex Projective VarietiesProjective Varieties

This is an introduction to algebraic geometry that I read as an undergraduate in an independent study seminar alongside the seniors in my doctoral program. At that time, I was awful at presentations, and the seniors in my study pointed out numerous mistakes every time I got up to speak. Nonetheless, I feel that I learned the right way to approach my studies in mathematics through this independent seminar. This book is exceptional in how it delivers proofs and definitions, showcasing the outstanding talent of the author (who is also a Fields Prize winner).

Fist of the North Star

In the manga, Kenshiro defeats Raoh, who yells "Of this life… I have not a single regret!" before ascending to heaven. Many people mistakenly believe that's the end of the story. In fact, that famous scene occurs at the beginning of volume 9 of the 15-volume series (in the paperback edition). The story continues well after Raoh's ascension, including a visit to the Kingdom of Shura, an appearance from Raoh's son, and more. I also like retro manga such as "Tomorrow's Joe" and "Black Jack," which I keep in my laboratory.

An introduction to Morse theory

Morse theory investigates manifolds by focusing on singularities (critical points) of functions and is the basis for obtaining Kirby diagrams. I read this book during the junior year of my undergraduate program, and it was this book that got me interested in topology.

Your Pronunciation Makes or Breaks your English Listening Skills! UDA Style 30-sound Practice Book

I spent 2 months in the U.S. when I was working on my master's degree. Much to my despair, no one could comprehend my English, which prompted me to buy this book as soon as I got back to Japan. I was scheduled to go to Germany for my half-year study abroad program in just 4 months, so while home I practiced my pronunciation like there was no tomorrow. I still haven't mastered everything in the book, but thanks to these exercises my English is slightly more comprehensible to locals. This book will make you realize truly how difficult English pronunciation is.

Getting a math paper published

Kenta Hayano

Few people are familiar with the process of how a research project in mathematics begins and or how it ends up published as a paper in an academic journal. Naturally, not all research follows one uniform path, so while I cannot speak for everyone's experience, I would like to describe the publication process for one of my papers.

This particular study was inspired by a question a colleague posed to me during a break at an academic conference in November, 2017. While I couldn't answer the question offhand, its simplicity led me to assume that I could find the answer after a quick review of existing research on the topic. That wasn't the case. After looking into a few things, it quickly became apparent that I did not know the answer and that I had completely underestimated how difficult the proposed problem actually was. My interest was piqued. After considerable time and effort, I was able to prove a theorem that addressed the problem in May 2018. I completed a paper summarizing my findings in September of the same year. Then came the challenge of getting my work published in an academic journal.

In general, to get your research published you need the journal's editors, and a panel of anonymous reviewers (called "peer reviewers") selected by the editors, to certify the quality of your work and formally accept your paper. With this particular paper, there was no real reason to rush publication, so I first tried my luck with a major journal. Given its standing though, I wasn't exactly shocked when my paper was rejected in December 2018. For my next step, I submitted my work to a slightly less prestigious journal. This was ultimately an incredible waste of time.

First off, I didn't hear back from them for an entire year. So in February 2020, I contacted them only to be told to "just keep waiting" because they "have asked the reviewers to check the paper but have yet to receive a response." Things weren't looking good at this point. It might have been better to withdraw my submission, but against better judgement, I decided to wait. The radio silence continued, so I followed up with the editors again in June 2020. This time they told me that they were looking for new peer reviewers. I was dumbfounded and irritated that they had the gall to start a search this late in the process. However, given how much time I had sunk into this submission, I decided to stick it out. I tried contacting them again in December 2020, but still didn't get a decent response. Finally, in March 2021, they informed me of their decision with a curt "We cannot accept your paper due to a backlog of papers awaiting publication." In other words, I was kept waiting for over two years for them to dismiss my paper on a whim. I vowed to never submit a paper to this journal ever again.

Still reeling from this rejection, I did some research before deciding my third target, and ended up submitting to Mathematica Scandinavica. I had a pretty good impression of this journal. They were transparent about their backlog, even offering the median number of days it would take to peer review papers accepted in the previous year. If any of my colleagues are reading this article, I strongly recommend you consider submitting your next paper to this journal.

At any rate, my discretion in selecting a journal paid off. In September 2021, I received revisions from the peer reviewers, promptly made the necessary edits, my paper was accepted the following month, and, at long last, my work was published in June 2022. All in all, it took four and a half years from when I started my research, and just over 4 years from when I completed my paper, to get my work published. Math papers are notorious for taking a long time to peer review, but 4 years is extremely rare. Experience really is the best teacher.

理 工 学 Information

The 23rd Annual Science and Technology Exhibition, KEIO TECHNO-MALL 2022

A Place for "Human Companionship" to Create New Collaborations - Chance and Challenge in an Era of Change

KEIO TECHNO-MALL is an event aimed at sharing research outcomes of Keio University's Undergraduate Faculty and Graduate School of Science and Technology with a large audience, as well as serving as a platform to facilitate fruitful encounters between government, industry, and academia. It is one of the largest science and engineering exhibitions organized by a Japanese university.

This year marked the return of the event taking place in person after a three-year hiatus. Both the 2020 and 2021 iterations were held online to prevent the spread of COVID-19. Researchers will be given space at exhibition booths to present their work. The event is structured to contribute to implementing their research in the real world, and urge innovation based on their creative findings. We hope to see you there!

Date and time: 10:00–18:00, Friday, December 2, 2022
Venue : Hall E2, Tokyo International Forum
Programming: Research exhibits of developed products and practical demonstrations, short presentations by researchers, sessions with special guests, and more.
For more details: www.kll.keio.ac.jp/ktm/

Editor's postscript

This issue featured a professor from the Department of Mathematics for the first time in six years. Associate Professor Hayano is quite the conversationalist and maintains a very active lifestyle! He completely blew away my (admittedly misguided) preconception that mathematicians are perpetually bound to their desks, quietly scribbling formulas all the time. For example, despite having no real experience in cue games, he hit things off with a billiard-loving patron at a bar, eventually forming a team and practicing their skills. I wasn't even surprised when he said they went on to compete in a tournament together.

We have another issue set to release this year. This time, we made a rare departure from Yagami Campus to do a photoshoot on Hiyoshi Campus. We hope you'll join us again for the next issue!

(Yurina Tomohisa)

Cover of current issue : A Kirby diagram representing a 4-manifold

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